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MATCHING OF ASYMPTOTIC APPROXIMATIONS IN THE PROBLEM OF THE SCATTERING OF ACOUSTIC WAVES BY AN ELASTIC SPHERICAL SHELL†

V. A. KOVALEV

Moscow

e-mail: vlad_koval@mail.ru

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The problem of the scattering of a steady plane acoustic wave by a spherical elastic shell is considered. A procedure is proposed for constructing an approximate solution, based on matching the expansions for different asymptotic models of the interaction of the shell with the acoustic medium. In the neighbourhood of zero frequency and thickness-resonance frequencies, long-wave low-frequency approximations of the equations of the theory of elasticity (the Kirchhoff–Love theory of shells or its refinement) and long-wave high-frequency approximations respectively are employed. Outside these neighbourhoods a flat-layer model is used. A comparison with the exact solution confirms that the proposed approach enables one to describe, with a uniform small error, the scattered pressure and the resonance components of the partial modes over a fairly wide frequency band. © 2002 Elsevier Science Ltd. All rights reserved.

Asymptotic models of the interaction of a shell with an acoustic medium are based on asymptotic approximations of the three-dimensional equations of the theory of elasticity, which constitute the modern dynamic theory of shells (see, for example, [1]). The classical Kirchhoff–Love theory of shells and its refinements can be related to the low-frequency approximations. In the high-frequency region there are two types of asymptotic approximations. The long-wave high-frequency approximation describes the oscillation of a shell at frequencies close to the thickness-resonance frequencies of tension or shear. For the short-wave high-frequency approximation a characteristic feature is the small effect of the curvature of the shell.

The presence of regions where the asymptotic approximations agree enables them to be matched to one another in order to obtain a uniform approximation over a wide frequency band. For example, by matching the solutions corresponding to the Kirchhoff–Love theory, the theory of long-wave high-frequency oscillations and the plane (antiplane) theory of elasticity one can completely approximate the dispersion curves for a cylindrical shell [2].

Below we present a procedure for matching the asymptotic approximations in the problem of the scattering of a plane acoustic wave by a spherical shell. Asymptotic models, developed on the basis of the above-mentioned approximations and which describe the interaction of the shell with an acoustic medium, are used. In the neighbourhood of zero frequency an asymptotic model is employed based on the refined Kirchhoff–Love theory of shells [3, 4], in the neighbourhood of the thickness-resonance frequencies the theory of long-wave high-frequency oscillations of a shell, immersed in a liquid [5] is used, and outside these neighbourhoods a model of the flat-layer type [6], corresponding to the short-wave high-frequency approximation is employed. As is shown below, the region of applicability of the latter model overlaps the regions of applicability of the first two in the low-frequency region and in the region of the thickness-resonance frequencies respectively, which enables us to construct a uniform approximate solution both for the scattered pressure, and for the resonance components of the partial modes. A comparison with the exact solution [7] confirms that the proposed method is highly effective.

1. FORMULATION OF THE PROBLEM

Consider the scattering of a steady plane acoustic wave by a spherical shell. The pressure is the incident wave has the form

$$p_i = p_0 \exp[-i(k\xi + \omega t)] \tag{1.1}$$

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where p_0 is a constant, which has the dimensions of pressure, k is the wave number, ω is the cyclic frequency, t is the time and ξ is a spatial coordinate directed opposite to the direction of motion of the wave. Suppose a and b are the outer and inner radii of the shell respectively, $h = (a - b)/2$ is the half-thickness of the shell and $R = (a + b)/2$ is the radius of the middle surface.

We will refer the shell to a spherical system of coordinates. Since the problem is axisymmetrical, all the quantities depend only on two coordinates (r, θ). We will introduce the dimensionless parameters

$$\varkappa = \rho/\rho_1, \quad \beta_i = c_i/c, \quad i = 1, 2; \quad \gamma = c_2/c_1, \quad k = \omega/c \tag{1.2}$$

where c_1 and c_2 are the velocities of the dilatation and distortion waves in the shell, respectively, ρ_1 is the density of the shell material, c is the velocity of sound in the fluid, and ρ is the fluid density. The pressure in the incident wave can be represented in the form [8]

$$p_i = p_0 \sum_{n=0}^{\infty} (-i)^n (2n+1) j_n(kr) P_n(\cos\theta) \tag{1.3}$$

In formula (1.3) j_n is the spherical Bessel function, P_n is a Legendre polynomial, and the time factor $\exp(-i\omega t)$ is omitted here and henceforth.

The scattered pressure can also be represented in the form of a series in Legendre polynomials

$$p_s = p_0 \sum_{n=0}^{\infty} (-i)^n (2n+1) B_n h_n^{(1)}(kr) P_n(\cos\theta) \tag{1.4}$$

where B_n are the required constants and h_n is the spherical Hankel function of the first kind. Representations (1.3) and (1.4) satisfy Helmholtz' equation for the excess pressure in the fluid, and the scattered pressure p_s satisfies the radiation condition as $r \to \infty$. The coefficients B_n are found from the contact problem for the equations describing the motion of the shell. The solution corresponding to the three-dimensional equations of the theory of elasticity were obtained earlier in [7].

We will further assume that the shell thickness is small, i.e. the thin-wall parameter $\eta = h/R$ is small, and we will consider the asymptotic models of the interaction of the shell with the fluid.

2. THE FLAT-LAYER MODEL

The model of a "dry" flat layer has been widely used (see, for example, [8]) for the approximate determination of the resonance frequencies. This approach is based on an analogy between peripheral waves, which occur in the shell when the acoustic pressure is scattered, and Lamb waves in the layer. For the case of a circular cylindrical shell an asymptotic model was constructed in [6], which is a development of the flat-layer model and enables the interaction between the shell and the fluid to be described, i.e. one can determine not only the resonance frequencies but also the scattered pressure and the forms of the resonance curves. In this paper we will apply this model to a spherical shell.

In the flat-layer type model the equations of the shell oscillations have the form

$$\begin{gathered} \Delta_p\varphi + \beta_1^{-2} k^2 R^2 \varphi = 0, \quad \Delta_p\psi + \beta_2^{-2} k^2 R^2 \psi = 0 \\ \Delta_p = \partial^2/\partial\zeta^2 + \partial^2/\partial\theta^2, \quad \zeta = r/R - 1 \end{gathered} \tag{2.1}$$

where φ and ψ are the Lamé potentials. The stress–strain characteristics of the state of the shell can be expressed in terms of these in the form

$$\begin{gathered} u_r = \frac{1}{R}\left(\frac{\partial\varphi}{\partial\zeta} + \frac{\partial\psi}{\partial\theta}\right), \quad \sigma_r = \rho_1 c^2 \frac{1}{R^2}\left[-k^2R^2\varphi + 2\beta_2^2\left(\frac{\partial^2\psi}{\partial\zeta\partial\theta} - \frac{\partial^2\varphi}{\partial\theta^2}\right)\right] \\ u_\theta = \frac{1}{R}\left(\frac{\partial\varphi}{\partial\theta} - \frac{\partial\psi}{\partial\zeta}\right), \quad \sigma_{r\theta} = \rho_1 c^2 \frac{1}{R^2}\left[k^2R^2\psi + 2\beta_2^2\left(\frac{\partial^2\varphi}{\partial\zeta\partial\theta} + \frac{\partial^2\psi}{\partial\theta^2}\right)\right] \end{gathered} \tag{2.2}$$

The change from the equations of the theory of elasticity in a spherical system of coordinates to Eqs (2.1) and (2.2) rests on the assumption

$$\partial/\partial\zeta \sim \partial/\partial\theta \sim \omega R/c_2 \sim \eta^{-1} \tag{2.3}$$

which define the short-wave oscillations of the shell, to which resonances of large numbers ($n \sim \eta^{-1}$) correspond. Conditions (2.3) enable us to retain only higher derivatives in the equations of the theory of elasticity and to freeze the radial coordinate r on the middle surface. On the front surfaces of the shell we have the boundary conditions

$$\begin{gathered}\sigma_r|_{\zeta=\eta} = -(p_i + p_s)|_{r=a}, \quad u_r|_{\zeta=\eta} = \frac{1}{\rho c^2 k^2}\frac{\partial}{\partial r}(p_i + p_s)|_{r=a}, \quad \sigma_{r\theta}|_{\zeta=\eta} = 0 \\ \sigma_r|_{\zeta=-\eta} = 0, \quad \sigma_{r\theta}|_{\zeta=-\eta} = 0\end{gathered} \tag{2.4}$$

The pressures p_i and p_s are given by relations (1.3) and (1.4) respectively.

We will expand the Lamé potentials in series in Legendre polynomials

$$\varphi(\zeta,\theta) = \sum_{n=0}^{\infty}\varphi_n(\zeta)P_n(\cos\theta), \quad \psi(\zeta,\theta) = \sum_{n=0}^{\infty}\frac{\psi_n(\zeta)}{n_1}\frac{\partial P_n(\cos\theta)}{\partial\theta}, \quad n_1 = n + \tfrac{1}{2} \tag{2.5}$$

Substituting expansions (1.3), (1.4) and (2.5) into Eqs (2.1), (2.2) and (2.4) and using the asymptotic formula

$$\partial^2 P_n(\cos\theta)/\partial\theta^2 \approx -n_1^2 P_n(\cos\theta), \quad n \gg 1 \tag{2.6}$$

we determined the unknown coefficients B_n

$$B_n = -\frac{d_1 j_n'(x) - \chi j_n(x)}{d_1 h_n^{(1)\prime}(x) - \chi h_n^{(1)}(x)}, \quad \chi = 2d_2 \varkappa k^3 R^3 \tag{2.7}$$

$$d_1 = 4D_s D_a, \quad d_2 = \alpha_1(\operatorname{ch}(\alpha_1\eta)\operatorname{ch}(\alpha_2\eta)D_s + \operatorname{sh}(\alpha_1\eta)\operatorname{sh}(\alpha_2\eta)D_a), \quad x = ka$$

$$D_s = \gamma_0^4\operatorname{ch}(\alpha_1\eta)\operatorname{sh}(\alpha_2\eta) - 4\beta_2^4 n_1^2\alpha_1\alpha_2\operatorname{sh}(\alpha_1\eta)\operatorname{ch}(\alpha_2\eta), \quad \gamma_0^2 = 2n_1^2\beta_2^2 - k^2R^2$$

$$D_a = \gamma_0^4\operatorname{sh}(\alpha_1\eta)\operatorname{ch}(\alpha_2\eta) - 4\beta_2^4 n_1^2\alpha_1\alpha_2\operatorname{ch}(\alpha_1\eta)\operatorname{sh}(\alpha_2\eta), \quad \alpha_i = \sqrt{n_1^2 - \beta_i^{-2}k^2R^2}, \quad i = 1, 2$$

3. THE KIRCHHOFF–LOVE THEORY AND ITS REFINEMENT

The Kirchhoff–Love theory and its refinements can be regarded as long-wave low-frequency approximations of the dynamic equations of the theory of elasticity. Consequently, these theories are suitable for approximating the exact solution in the neighbourhood of zero frequency. In this paper we use the previously proposed [3] asymptotic model of the interaction of the shell with the fluid, which is based on the refined Kirchhoff–Love theory. The application of this model to a spherical shell was considered in [4].

Using the results obtained in [4], we will write the equations of motion of the shell in terms of displacements

$$\begin{gathered}\frac{\partial}{\partial\theta}\Phi(u) + (1-\nu)u + (1+\nu)\frac{\partial w}{\partial\theta} - \frac{1}{3}\eta^2\left[\frac{\partial}{\partial\theta}\Phi(\gamma_1) + (1-\nu)\gamma_1\right] + \\ + \frac{1-\nu}{2}R^2\frac{\omega_{tg}^2}{c_2^2}u + \underline{\frac{\nu(1+\nu)R}{2E}\frac{\partial m}{\partial\theta}} = 0 \\ (1+\nu)(\Phi(u) + 2w) + \frac{1}{3}\eta^2[\Delta_0\Phi(\gamma_1) + (1-\nu)\Phi(\gamma_1)] - \\ - \frac{1-\nu}{2}R^2\frac{\omega_{tr}^2}{c_2^2}w + \underline{\frac{\nu(1+\nu)R}{E}m + \frac{(1-\nu^2)R^2}{2Eh}Z} = 0\end{gathered} \tag{3.1}$$

where

$$k_0 = \operatorname{ctg}\theta,\quad \Delta_0 = \frac{\partial^2}{\partial\theta^2} + k_0\frac{\partial}{\partial\theta},\quad \Phi(f) = \frac{\partial f}{\partial\theta} + k_0 f,\quad \gamma_1 = \frac{\partial w}{\partial\theta} - u$$

u is the tangential displacement in the direction of the θ axis, w is the normal displacement (the sag), E is Young's modulus and ν is Poisson's ratio. The quantities Z, m and the reduced frequencies $\omega_{\rm tg}$ and $\omega_{\rm tr}$ are defined as follows:

$$Z = \left(1 \underline{- \frac{8-3\nu}{10(1-\nu)}\eta^2\Delta_0}\right)(p_i + p_s)\Big|_{r=a},\quad m = -(p_i + p_s)\Big|_{r=a} \tag{3.2}$$

$$\omega_{\rm tg}^2 u = \omega^2\left[u + \underline{\eta^2(B_{00} + B_{01}z^2 + B_{02}z^4)\frac{\partial}{\partial\theta}\Phi(u)}\right],\quad z = \frac{\omega h}{c_2} \tag{3.3}$$

$$\omega_{\rm tr}^2 w = \omega^2[w + \underline{\eta^2 A_{00}\Delta_0 w - z^2(A_{01}w + \eta^2 A_{02}\Delta_0 w)}] \tag{3.4}$$

$$B_{00} = -\frac{\nu^2}{3(1-\nu)^2},\quad B_{01} = -\frac{\nu^2(3-5\nu-\nu^2)}{45(1-\nu)^3},\quad B_{02} = \frac{\nu^2(-17+56\nu-33\nu^2-28\nu^3+5\nu^4)}{1260(1-\nu)^4}$$

$$A_{00} = \frac{7\nu-17}{15(1-\nu)},\quad A_{01} = \frac{422-424\nu-33\nu^2}{1050(1-\nu)},\quad A_{02} = \frac{32-96\nu+261\nu^2-197\nu^3}{15750(1-\nu)^2}$$

The impermeability condition has the form

$$w + \underline{\frac{\nu\eta^2}{2(1-\nu)}\Delta_0 w - \frac{\nu\eta}{(1-\nu)}\Phi(u)} = \frac{1}{\rho\omega^2}\frac{\partial}{\partial r}(p_i + p_s)\Big|_{r=a} \tag{3.5}$$

The underlined terms in these equations enable us to take into account the transverse squeezing of the shell and certain other phenomena (for more detail see [3, 4]). Neglecting these, we arrive at a model of the interaction of the shell with the fluid, based on the classical Kirchhoff–Love theory. The quantities, p_i and p_s, as previously, are defined by relations (1.3) and (1.4).

The following relation for the unknown coefficients B_n were obtained in [4]

$$B_n = -\frac{d_1 j_n'(x) + \chi j_n(x)}{d_1 h_n^{(1)\prime}(x) + \chi h_n^{(1)}(x)},\quad \chi = \frac{\varkappa kR}{2(1+\nu)}\beta_2^{-2}(b_2 d_2 + b_1 d_3) \tag{3.6}$$

$$d_1 = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix},\quad d_2 = \begin{vmatrix} a_{12} & a_{11} \\ a_{32} & a_{31} \end{vmatrix},\quad d_3 = \begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$a_{11} = 1 + \nu + \frac{1}{3}\eta^2 s,\quad a_{12} = -\left(1 + \frac{1}{3}\eta^2\right)s + \frac{1-\nu}{2}R^2\frac{\omega_{\rm tg}^2}{c_2^2}$$

$$a_{21} = 2(1+\nu) + \frac{1}{3}\eta^2 N s - \frac{1-\nu}{2}R^2\frac{\omega_{\rm tr}^2}{c_2^2},\quad a_{22} = -Na_{11}$$

$$a_{31} = 1 - \frac{\nu\eta^2}{2(1-\nu)}N,\quad a_{32} = \frac{\nu\eta}{(1-\nu)}N,\quad b_1 = \frac{\nu(1+\nu)}{2}$$

$$b_2 = \nu(1+\nu) - \frac{1-\nu^2}{2\eta}\left(1 + \frac{8-3\nu}{10(1-\nu)}\eta^2 N\right),\quad N = n(n+1),\quad s = N - 1 + \nu$$

The regions in which the refined asymptotic model and the model based on the classical Kirchhoff–Love theory can be used are limited by the inequalities

$$\omega R / c_2 \ll \eta^{-1},\quad \omega R / c_2 \ll \eta^{-1/2} \tag{3.7}$$

respectively (see [3]). Hence, these models are only suitable for describing resonances of the wave A

generated by the fluid and resonances of Lamb type waves S_0 and A_0 with numbers $n \ll \eta^{-1}$ (the refined Kirchhoff–Love theory) and $n \ll \eta^{-1/2}$ (the classical Kirchhoff–Love theory).

4. THE LONG-WAVE HIGH-FREQUENCY APPROXIMATIONS

The first higher-order Lamb-type wave resonances correspond to long-wave high-frequency oscillations of the shell. An asymptotic theory of long-wave high-frequency oscillations of a shell immersed in a liquid was constructed in [1, 5]. In this theory two types of long-wave high-frequency approximations are distinguished. The transverse approximation is used in the neighbourhood of tension-compression thickness-resonance frequencies, i.e. $|z - \Lambda_{st}| \ll 1$, where z is given by formula (3.3), $\Lambda_{st} = \pi m/\gamma$ (for antisymmetric modes) and $\Lambda_{st} = \pi(m - 1/2)/\gamma$ (for symmetric modes), $m = 1, 2 \ldots$. The resolvent of the transverse approximation has the form

$$\eta^2(T\Delta_0 w - 8w) + (z^2 - \Lambda_{st}^2)w = \pm\frac{(-1)^m h}{\rho_1 c_2^2}(p_i + p_s)(1+\eta) \tag{4.1}$$

$$T = \frac{1}{\gamma^2} \mp \frac{8}{\Lambda_{st}}\begin{Bmatrix}\operatorname{tg}\Lambda_{st}\\ \operatorname{ctg}\Lambda_{st}\end{Bmatrix} \tag{4.2}$$

Here and henceforth the upper (lower) sign and the upper (lower) expression in the braces correspond to the antisymmetric (symmetric) modes.

The impermeability condition is written as follows:

$$\pm(-1)^m w(1-\eta) = \frac{1}{\rho c^2 k^2}\frac{\partial}{\partial r}(p_i + p_s)\Big|_{r=a} \tag{4.3}$$

In the neighbourhood of the shear thickness-resonance frequencies the tangential approximation is used. In this case the resolvent and the impermeability condition take the form

$$\eta^2\left(P\frac{\partial}{\partial\theta}(\Phi(u)) - 4u\right) + (z^2 - \Lambda_{sh}^2)u = \frac{2(-1)^{m+1}h\gamma}{\Lambda_{sh}\rho_1 c_2^2}\eta\frac{\partial(p_i + p_s)}{\partial\theta}\begin{Bmatrix}\operatorname{ctg}(\gamma\Lambda_{sh})\\ \operatorname{tg}(\gamma\Lambda_{sh})\end{Bmatrix} \tag{4.4}$$

$$\frac{2(-1)^m\gamma}{\Lambda_{sh}}\eta\Phi(u)\begin{Bmatrix}\operatorname{ctg}(\gamma\Lambda_{sh})\\ \operatorname{tg}(\gamma\Lambda_{sh})\end{Bmatrix} = \frac{1}{\rho c^2 k^2}\frac{\partial}{\partial r}(p_i + p_s)\Big|_{r=a},\quad P = 1 \pm \frac{8\gamma}{\Lambda_{sh}}\begin{Bmatrix}\operatorname{ctg}(\gamma\Lambda_{sh})\\ \operatorname{tg}(\gamma\Lambda_{sh})\end{Bmatrix} \tag{4.5}$$

In Eqs (4.4) and (4.5) $\Lambda_{sh} = \pi(2m - 1)/2$ (for antisymmetric modes) and $\Lambda_{sh} = \pi m$ (for symmetric modes); $m = 1, 2 \ldots$. They are used when $|z - \Lambda_{sh}| \ll 1$.

In Eqs (4.1) and (4.3)–(4.5) the quantities p_i and p_s, as before, are defined by expansions (1.3) and (1.4). Consider the case of the antisymmetric tangential approximation. We expand the displacement u in series in Legendre polynomials

$$u = \sum_{n=0}^{\infty} C_n\frac{\partial P_n(\cos\theta)}{\partial\theta} \tag{4.6}$$

and substitute expansions (1.3), (1.4) and (4.6) into (4.4) and (4.5), Using the relations

$$\Delta_0 P_n(\cos\theta) = -n(n+1)P_n(\cos\theta),\quad \Phi\left(\frac{\partial}{\partial\theta}P_n(\cos\theta)\right) = \Delta_0 P_n(\cos\theta) \tag{4.7}$$

we obtain a system of linear algebraic equations, from which we determine the unknown coefficients B_n. They have the form

$$B_n = -\frac{Sj_n'(x) - Qj_n(x)}{Sh_n^{(1)\prime}(x) - Qh_n^{(1)}(x)} \tag{4.8}$$

$$S = -Pn(n+1) - 4 + \eta^{-2}(z^2 - \Lambda_{sh}^2),\quad Q = 4n(n+1)h\varkappa k\beta_1^{-2}\operatorname{ctg}^2(\gamma\Lambda_{sh})\Lambda_{sh}^{-2}$$

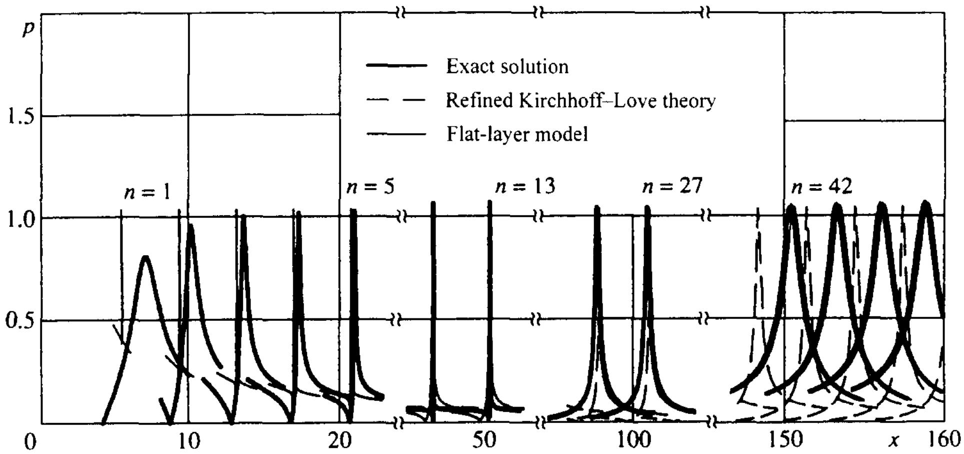

Fig. 1

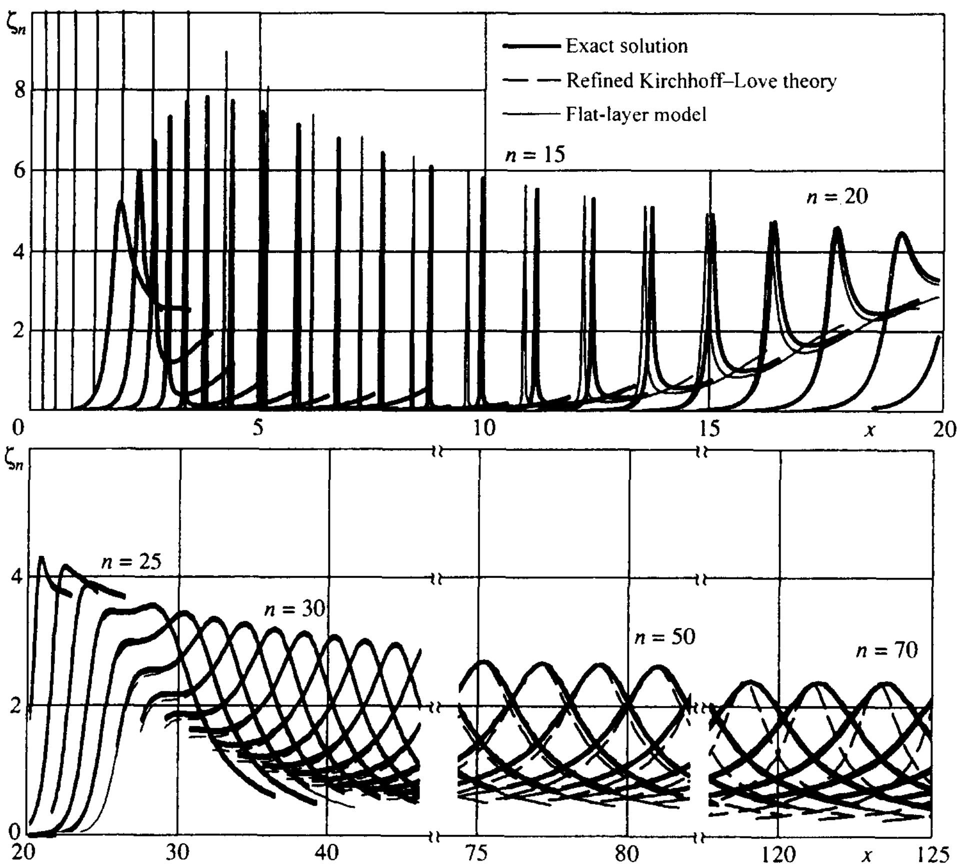

Fig. 2

Note that solution (4.8) and similar solutions for other types of long-wave high-frequency approximations are only applicable for small values of the parameter n ($n \ll \eta^{-1}$). But series (1.4) only begins to converge when $n \sim x \sim \eta^{-1}$ (see [8]), i.e. the solution contains only short-wave components. Consequently, when calculating the scattered pressure using formula (1.4) the long-wave high-frequency approximations must be used together with the flat-layer model.

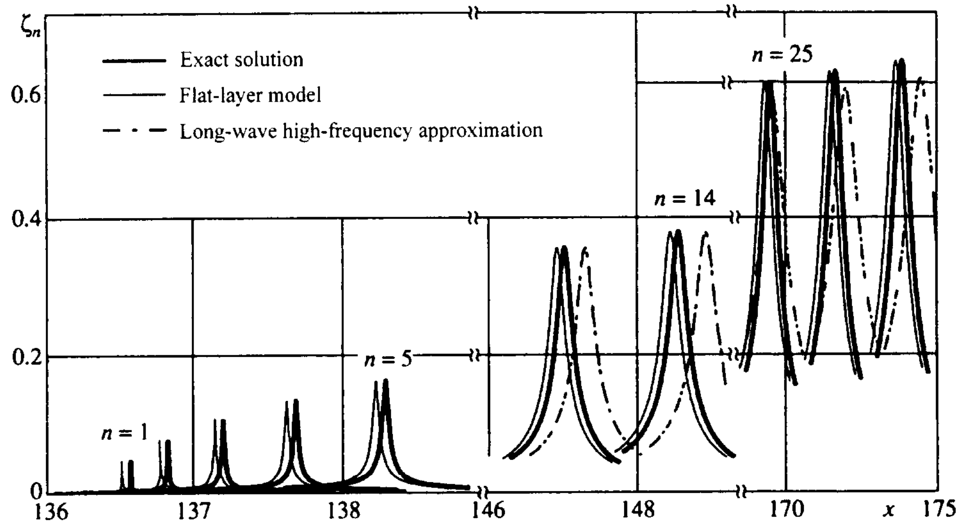

Fig. 3

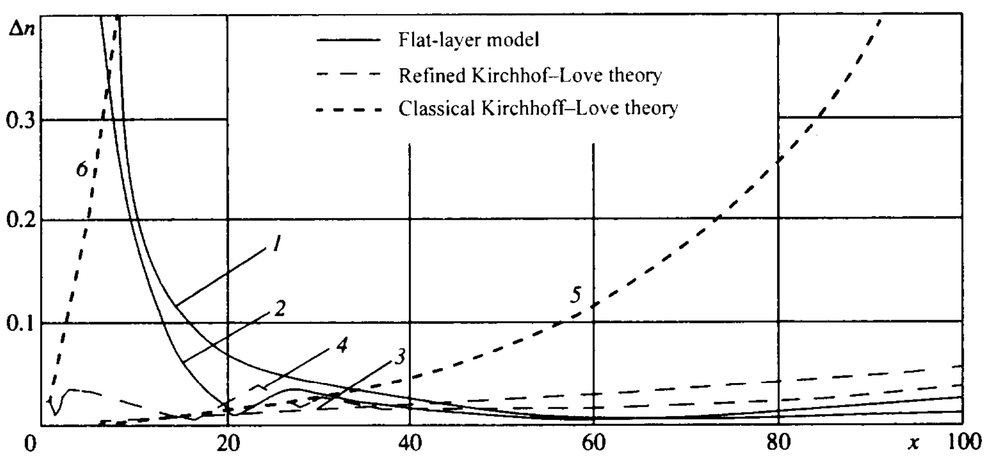

Fig. 4

5. MATCHING OF THE ASYMPTOTIC EXPANSIONS

We will consider the matching of the asymptotic expansions described above. We begin by investigating the resonance components of the partial modes using the strict basis [8]

$$\zeta_n = \frac{2(2n+1)}{x}\left|B_n + \frac{j_n'(x)}{h_n^{(1)\prime}(x)}\right| \tag{5.1}$$

to distinguish the resonances.

In Fig. 1 we show the resonance components for the Lamb-type wave S_0, calculated using the refined Kirchhoff–Love theory (the dashed curve) and the flat-layer model (the continuous curve). The coefficients B_n in formula (5.1) were found either from formula (3.6) for the refined Kirchhoff–Love theory, or from formula (2.7) for the flat-layer model. The calculations were carried out for the following values of the problem parameters

$$c_1 = 5960\ \text{m/s},\quad c_2 = 3240\ \text{m/s},\qquad c = 1493\ \text{m/s}$$

$$\rho_1 = 7700\ \text{kg/m}^3,\quad \rho = 1000\ \text{kg/m}^3,\qquad \eta = 1/39 \tag{5.2}$$

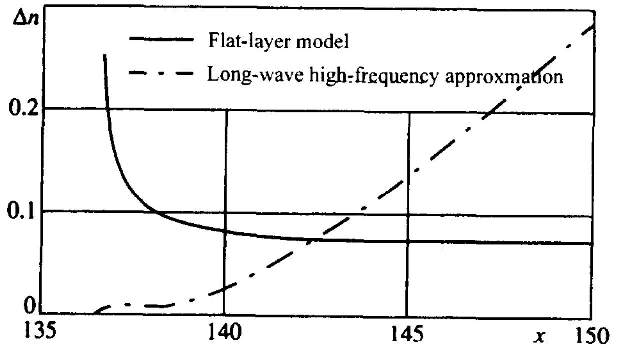

Fig. 5

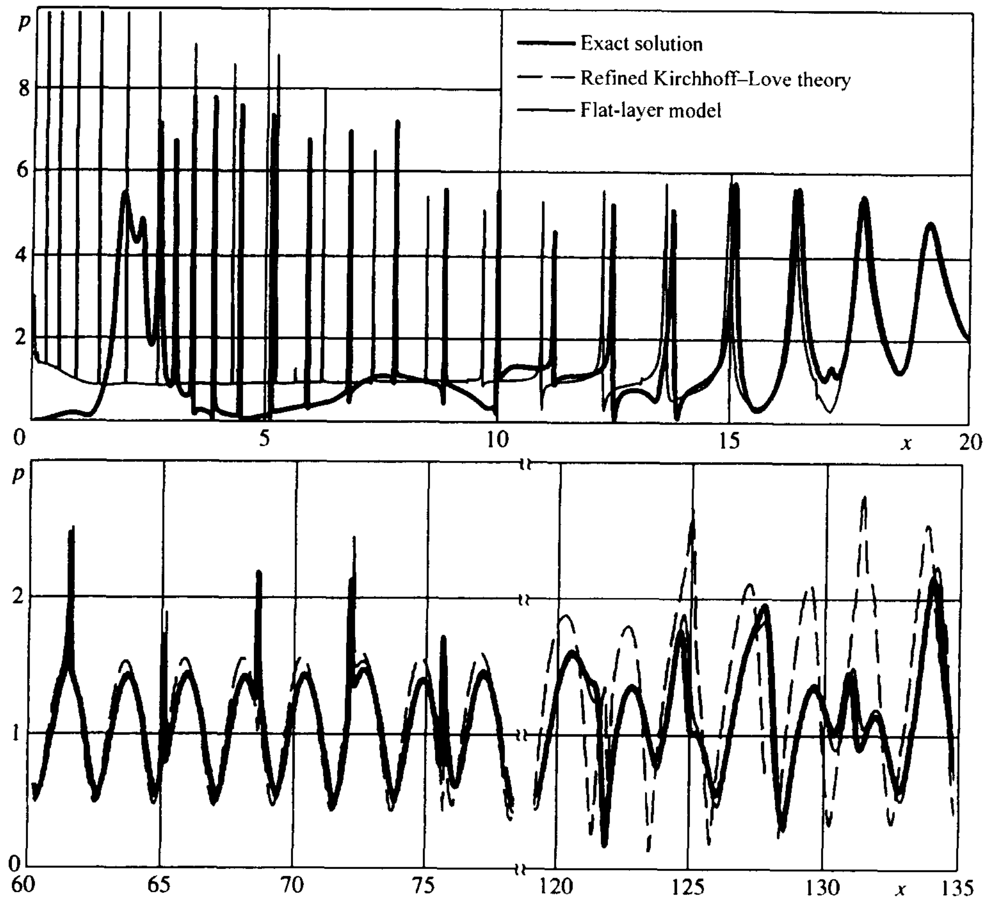

Fig. 6

The exact solution, obtained using the three-dimensional equations of the theory of elasticity (see [7]), are also shown in Fig. 1 and are denoted by the continuous heavy curve. A similar comparison of the resonance components is shown in Fig. 2 for the wave A generated by the fluid (beginning with $n = 25$ it is replaced by a Lamb-type wave A_0).

In Fig. 3 we compare the resonance components for a Lamb-type wave A_1, corresponding the flat-layer model (the continuous curve), the long-wave high-frequency approximation (the dash–dot curve) and the exact solution (the continuous heavy curve). For the long-wave high-frequency approximation the coefficients B_n are given by formula (4.8).

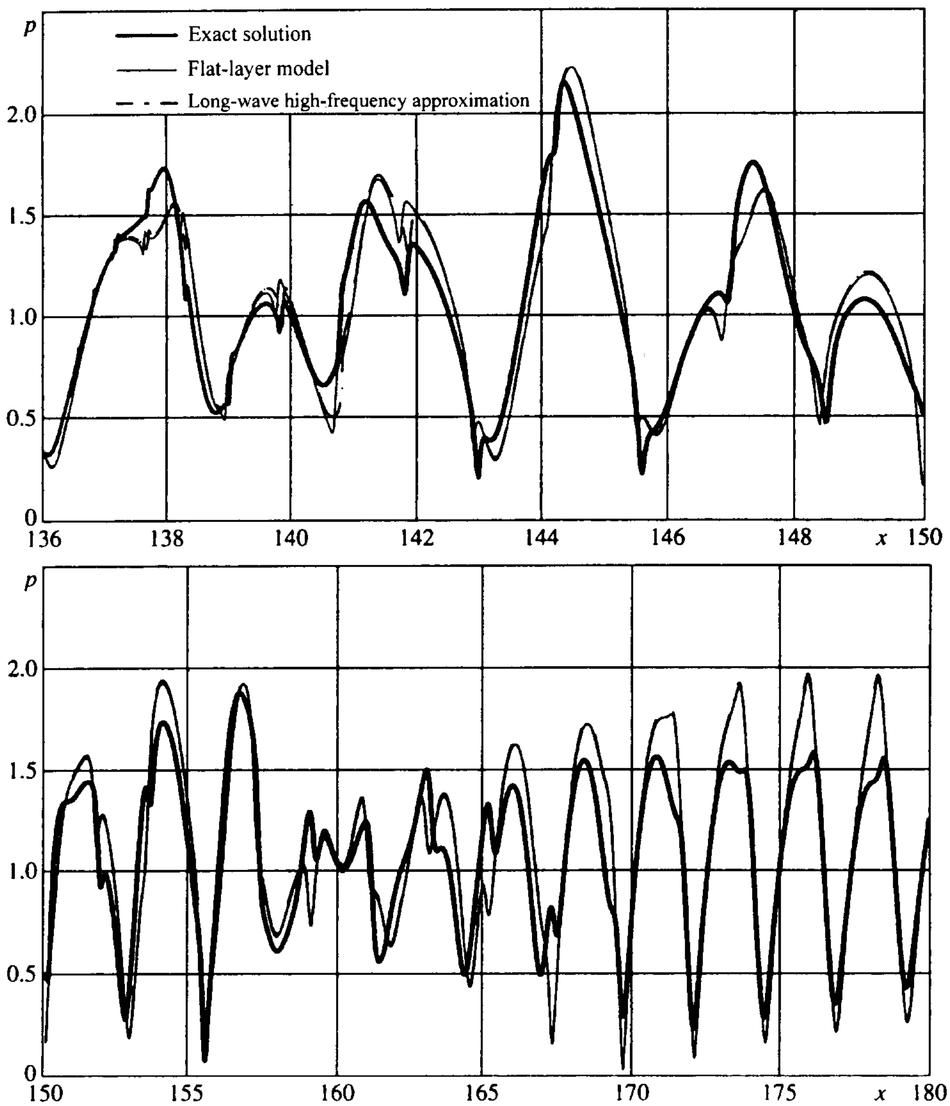

Fig. 7

In Figs 4 and 5 we show the errors of the approximation of the dispersion curve. Here

$$\Delta n = \left| n^{\mathrm{app}} - n^{\mathrm{ex}} \right| \tag{5.3}$$

where n^{ex} and n^{app} are the exact and approximate values of the wave number. In Fig. 4 the values of n^{app} for the S_0 wave correspond to the odd numbers of the curves, and for A and A_0 waves they correspond to the even numbers of the curves. In Fig. 4 curves 1 and 2, calculated for the exact Kirchhoff–Love theory, are shown by the thin dashed lines, curves 3 and 4, for the classical Kirchhoff–Love theory, are shown by the thick dashed curves, and curves 5 and 6, obtained for the flat-layer model, are shown by the continuous curves. In Fig. 5 we show the errors of the approximation of the dispersion curves Δn for the A_1 wave, where the values of n^{app}, obtained for the flat-layer model, are represented by the continuous curves and the data calculated for the long-wave high-frequency approximation are represented by the dash–dot curve.

Figures 1–5 illustrate the existence of regions in which the asymptotic models considered are matched. The results of synthesizing the form function of the scattered pressure [8]

$$p = \frac{2}{x} \left| \sum_{n=0}^{\infty} (-1)^n (2n+1) B_n \right| \tag{5.4}$$

in the far field ($r \to \infty$) in the case of backward scattering ($\theta = 0$) are shown in Figs 6 and 7. Data for the region where the values of the external wave radius x lie in the range (20, 60) are not shown in Fig. 6. In this region both the refined Kirchhoff–Love theory and the flat-layer model describe the form function of the scattered pressure p quite accurately. In Fig. 7, when making calculations using the long-wave high-frequency approximation, solution (4.8) was only used for $n < 10$. To calculate the remaining terms of series (5.4) we used the flat-layer model. This scheme enables the first resonances of the Lamb-type wave A_1 to be described exactly.

The results presented in Figs 1–7 show that the approach proposed in this paper enables the solution of the scattering problem to be approximated quite accurately. We also note that the refined Kirchhoff–Love theory, without being in any way inferior in simplicity to the classical theory, has a much wider range of application.

As is well known, an exact solution of the scattering problem only exists for spherical and circular cylindrical shells. The approach developed above can be extended to shells of more complex shape, for which there is no exact solution.

REFERENCES

1. KAPLUNOV, J. D., KOSSOVICH, L. Yu. and NOLDE, E. V., *Dynamics of Thin Walled Elastic Bodies*. Academic Press, New York, 1998.
2. BEREZIN, V. L., KOSSOVICH, L. Yu. and KAPLUNOV, J. D., Synthesis of the dispersion curves for a cylindrical shell on the basis of approximate theories. *Sound Vibr.*, 1995, **186**, 1, 37–53.
3. BELOV, A. V., KAPLUNOV, J. D. and NOLDE, E. V., A refined asymptotic model of fluids-structure interaction in scattering by elastic shells. *Flow, Turbul. Combust.*, 1999, **61**, 1–4, 255–267.
4. KOVALEV, V. A., The use of a refined asymptotic model in the problem of the scattering of a plane acoustic wave by a spherical shell. *Izv. Ross. Akad. Nauk. MTT*, 2002, 2, 155–162.
5. KAPLUNOV, J. D., High-frequency stress–strain states of small variability in shells immersed in a liquid. *Prikl. Mat. Mekh.*, 1991, **55**, 3, 478–485.
6. VIL'DE, M. V., KAPLUNOV, J. D. and KOVALEV, V. A., The development of the flat-layer type approximation in the problem of the scattering of acoustic waves by a cylindrical shell. *Izv. Ross. Akad. Nauk. MTT*, 2002, 3, 180–186.
7. GOODMAN, R. R. and STERN, R., Reflection and transmission of sound by elastic spherical shells. *J. Acoust. Soc. America*, 1962, **34**, 3, 338–344.
8. VEKSLER, N. D., *Acoustic Spectroscopy*. Valgus, Tallinn, 1989.

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